Induction coil guns for hypervelocities

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Abstract: The tubular linear induction motor is seen as an alternative to rail guns, whose maximum speed is limited by current collection problems. By reducing the task of determining the maximum speeds attainable by induction to first-order approximations, the author shows that the ultimate limitations are set by the physical properties of copper; specific heat, resistivity, density, melting point and yield stress. The speeds attainable under these constraints are then calculated in terms of track length and missile mass.

1 Introduction

Electromagnetic launch technology during the 1970s and 1980s was largely concentrated on rail guns, that is to say devices in which the moving conductor slides between a pair of conducting rails from which it collects direct current as it is accelerated. At the electromagnetic launch conference held in London in 1991 [1] opinion seemed to indicate that 7 to 8 km/s was the highest velocity that could be achieved by rail guns. Destruction of the track by burning as a result of arcing at the sliding contacts was the principal limitation.

The alternative type of launcher is the induction accelerator, requiring no electrical connection to the moving member. The most popular of these is the tubular type, in which the primary consists simply of a row of coaxial coils. The name 'coil gun' has been adopted to describe this class of linear induction motor.

International interest in launchers appears to be focused on two main aspects; the attainment of velocities of the order of 6.0 km/s in large masses and the attainment of much higher velocities in masses perhaps as small as 2 g. Elliott [2] gives a detailed design for a tubular launcher for a mass of 2 kg to reach 3 km/s in a distance of only 5 m. Driga and Weldon [3] give a design for a multistage coil gun to accelerate a 14 kg mass to 6.0 km/s in a distance of 192 m. Sandia National Laboratories at Alberquerque, New Mexico, report experimental work on masses of only 11 g [4]. NASA reports include even more ambitious targets for earth-to-space launchers requiring velocities of 11 km/s [5]. The requirements for large and small masses call for two very different approaches.

A companion paper [6] deals with the large-mass application where one solution emerges as a double-sided sandwich motor with no iron core. Although the target velocity for this particular load did not reach the frontiers of what was possible, there were signs that pointed to the steps to be taken in the event of that particular topology failing, one of which was to be found in the value of the end-winding factor.

As field velocities are increased, a fairly obvious limitation is likely to be leakage reactance, which in turn involves skin effect. There comes a point therefore where further increase in velocity must be achieved by increasing pole pitch rather than by increasing frequency. In conventional machines this involves increasing the length of the end windings and hence their resistance and leakage reactance. In the companion paper [6] the target speed was 1.2 km/s, and even using a new technique of diamond-shaped surface windings, where the end windings themselves are seen to contribute to the working thrust, the pole pitch had to be increased to four times the width of the machine. The end-winding factor, being the ratio of the long side of the diamond to the semi-width, had thus risen to the value of $\sqrt{17}$. There is no doubt therefore that for velocities higher than 1200 m/s the tubular motor with its total absence of end conductors is the obvious choice.

References 2 to 5 all propose the use of tubular motors. In particular, Reference 4 claims to have 'demonstrated orthogonal coil technology (AC) at 1 km/s and 160 g'. Launchers for such velocities clearly involve very high values of acceleration and therefore would appear to involve transient behaviour of induction motors. The combination of an accelerated field system and time-transient operation suggests that almost any but the most complex theoretical approach is unlikely to achieve any kind of accuracy.

The approach adopted here aims to make at least a first-order approximation to what might be expected, since the very nature of the problem of obtaining maximum possible speeds involves the use of parameters that lend themselves to simplified treatment. For example, it is proposed to use secondary conductor so thin that skin effect is virtually eliminated, for the obvious reason that to involve secondary skin effect is to include relatively inactive mass in the secondary which reduces the acceleration. Attaining maximum acceleration is one of the main objectives in this type of launcher.

Skin effect could be said to be another way of describing secondary leakage effects. In the absence of skin effects the secondary appears as a pure resistance and there is zero secondary time constant.

Short-secondary machines must essentially have parallel-connected primaries so that the flux is forced at all points [7]. By maintaining a condition in which the secondary conductor has a limited effect on the current drawn from the supply, the effect of primary leakage reactance is limited to the importance of the time needed to attain the necessary flux prior to the arrival of the missile.

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Finally, by making the secondary acceptably long, transient secondary edge effects are also eliminated [8].

2 Basic tubular motor: the 'Jumping Ring'

In tubular motor technology there is nothing simpler than a single-coil primary and a single-turn short circuited secondary. This primitive motor, known as the 'Jumping Ring' and shown diagrammatically in Fig. 1, had its origins in 1882, six years before Tesla's invention of the rotating induction motor.

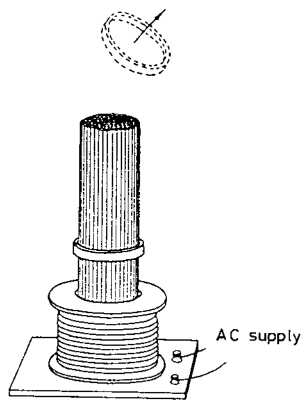

Fig. 1 *The 'Jumping Ring'*

Both primary coil and secondary conducting ring are threaded on an extended iron core. The magnetic circuit of such a system is remarkably good, being equivalent to a closed iron ring with only a very small airgap [9]. When an alternating EMF is applied to the primary with the secondary ring lying on top of it, the latter experiences a very large repulsive force. Because of the quality of the magnetic circuit it is easy to apply sufficient current to saturate the iron core. At this point one obtains virtually the maximum steady-state force on the secondary ring and the height at which it floats cannot be increased substantially by increasing the voltage. But transiently the situation is quite different.

In the case of one particular example of this apparatus, an applied voltage that was sufficient to saturate the core was found to eject the ring vertically to a height of 4 m. The applied voltage was then increased some six times and the ring was ejected to over 20 m. But then the accelerating force has been exerted during the first fraction of the first halfcycle of the supply. The induced EMF was solely the result of *rate of change* of flux (as, of course, Faraday originally pointed out) and the secondary was never concerned with the subsequent saturation of the core and the huge primary current that then flowed. It is not difficult to take the next step in the jumping ring sequence and remove the iron core.

Physical theory predicts that the force between oppositely flowing currents in infinitely thin wires is inversely proportional to their separation and that therefore when in contact the force is infinite. What happens to the flux in such circumstances is largely speculative, indeed preoccupation with where the flux goes in such a topological situation can be a very unprofitable exercise.

Experimentally, this simple form of a single-layer 'pancake' primary coil and a thin foil secondary was used by Russian experimenters, who in 1977 claimed to have achieved 4.9 km/s from rest in a distance of 3 mm [10], representing an average acceleration of 4×10^8 g. On the face of it, continued acceleration at this rate would achieve 20 km/s in 5 cm from rest. But, of course, the foil ring was vapourised. The levels of current and flux densities during the action can hardly be imagined. And 3 mm virtually took the secondary out of range of the primary, which is why single-coil and single-turn secondaries are not necessarily acceptable as coil guns, for the secondary must pass through a series of primary coils and the secondary is only tightly coupled magnetically when it is exactly opposite a primary coil. Such considerations lead directly to the important question of the 'goodness factor' [11] of a tubular motor.

3 Tubular motor theory

In conventional induction motors the gap flux from one half of a pole passes through the core. In short primary machines, however, especially in those with parallel connected primaries, standing waves are known to occur [7] and it must therefore be assumed, for the worst case, that the relationship between the pole, or gap, peak flux density B_g and the core flux density B_c is the result of considering the whole of the flux from one pole of the primary $(2\pi rp)$ B_g to pass through the core whose maximum sectional area πr^2 contains B_c. Thus

$$B_g = B_c(r/2p)$$

where p is the pole pitch. The mean length of the portion of the magnetic circuit along the core of a ironless motor is a pole pitch, its area basically πr^2, giving it a reluctance $p/\pi r^2 \mu_0$. The only bonus in using a tubular motor is that the reluctance outside the primary is relatively small in the absence of any iron, varying from an equivalent airgap of p/π, when the ratio of r/p is large, to virtually zero provided r/p is small. Fig. 2, taken from Reference 8,

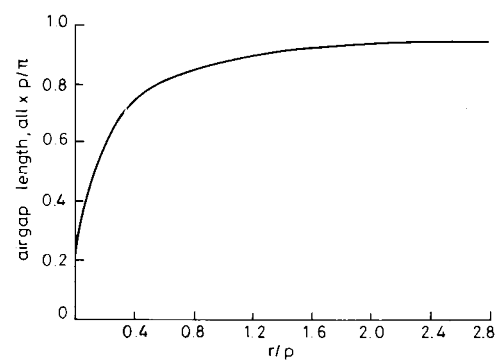

Fig. 2 *Airgap length against r/p*

shows the reduction factor on p/π plotted as a function of r/p.

The resistance of the electric circuit depends on the thickness of the conduction tube used. Here, a compromise is needed between too thin a conducting wall to give a low enough resistance, and too thick a wall to allow enough flux to penetrate to the central core so as to embrace all the secondary conductor. If the central conductor is of thickness t, which is small compared to r, the length of the electric circuit is $2\pi r$ and its sectional area is pt, so its resistance is $\rho_c(2\pi r/pt)$. Thus, assuming all the reluctance to be in the core, the goodness factor is

$$G = [p\omega/(p/\pi r^2 \mu_0)(\rho_c 2\pi r/pt)]$$

This is seen to simplify to

$$G = (\omega\mu_0/\rho_c)rt/2 \quad (1)$$

but this is only true provided both t and the thickness of the primary winding are small compared with both r and p.

Other factors that would also need to be considered in detail include:

(*a*) Skin effect. The thickness of the secondary tube wall at the high-speed end of the track should not exceed the skin depth.

$$\delta = \sqrt{(2\rho_{hot}/s\omega\mu_0)} \quad (2)$$

where s is the fractional slip and ρ_{hot} is the resistivity of copper just before it melts.

(*b*) Leakage reactance could prove a serious constraint on design. It may well impose a limitation on wall thickness and certainly on any radial space between primary and secondary.

4 Secondary heating in induction accelerators

In the case of a conventional tubular motor with a pure travelling field, the basic equation for all induction motors applies, namely

fractional slip = (secondary loss)/(force × sync. speed)

Thus the secondary loss in accelerating from speed v_0 to synchronous speed v_s is equal to $m[v_c v_0 - v_0^2/2]$, a formula derived in the companion paper [6]. Thus in accelerating from rest up to synchronous speed against no load other than its own inertia, the heat loss in the secondary is equal to its full speed kinetic energy $mv^2/2$. Doing an energy balance for a secondary of mass m, specific heat S and melting point T°C, the maximum speed v_m that can be attained without melting is given by $mv_m^2/2 = mST$. For copper, $S = 385$ J/kg°C, $T = 1053$°C, so that $v_m = 900$ m/s and this result is independent of the mass of the secondary.

It is clear at once that fixed-field speeds are useless for high-velocity accelerators. The field must be graded in velocity so that the secondary always finds itself in a field that is travelling only slightly faster than itself. If, in a graded field system, the slip is always maintained at a value s, then it can be shown that the equation is modified to $smv_m^2/2 = mST$, or

$$s = 2ST/(v_m)^2 \quad (3)$$

At first sight this presents no apparent problem. It is surely possible to grade the field so that the slip is never more than 1%, which would raise v_m to 9000 m/s, or even higher for lower values of slip. Actually, it raises a new and more obscure issue.

If the track length is to be limited, demands for higher velocities require greater acceleration of the field. This soon becomes so high that the difference in slip between one end of the missile and the other is greater than the permitted value given in eqn. 3. Shrinking the length of the missile is only possible until its length, in relation to the pole pitch, brings it into the region where transient short-secondary effects become prohibitive [8].

For example, consider a tubular motor primary 100 m long, required to accelerate a secondary to 10 000 m/s. For constant acceleration this requires 500 000 m/s² so that the rate of increase of veocity with *distance* at any point along the track is 500 000 divided by the velocity at that point. This, as a fraction of the speed at the same point is $500\,000/v^2$/m, so that a secondary 20 cm long would experience 10% slip at its one extremity at the same time as the other was at zero slip, when its speed was 1000 m/s. To limit this even to 1% would involve a missile only 2.0 cm long. (Eqn. 3 suggests that the working slip should only be 1% for 9000 m/s.) One could accept that one might be able to achieve the first 1000 m/s in some other way (by chemical propellant, for example) but this is about as low as one could go with the initial velocity without reducing the secondary length to apparently impossible proportions, so here is the first conflict of requirements.

For now edge effects in linear motors are involved, particularly in those relating to short-secondary machines. The problem is complex. Basically, the phenomenon is illustrated in Fig. 3 which shows the

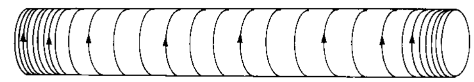

Fig. 3 *Current distribution in a solid cylinder of secondary conductor which is short in relation to pole pitch*

current distribution in a solid cylinder of secondary conductor which is short in relation to pole pitch. The current density builds up at each end owing to the edge transients. Fig 4 shows, effectively, the heat loss plotted as a function of length/pole pitch.

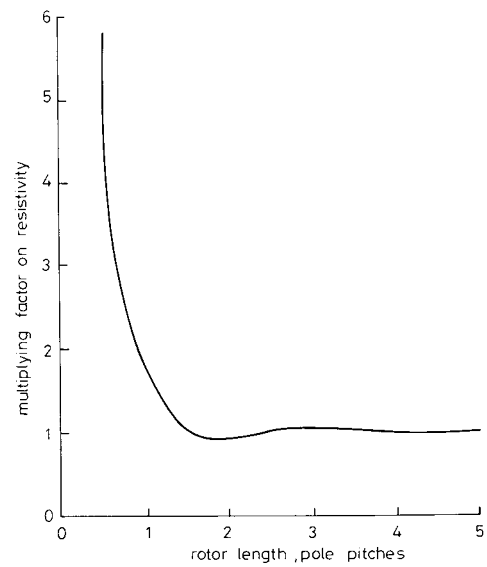

Fig. 4 *Heat loss as a function of length/pole pitch*

Since the values of slip involved are of the order of 1% or less, it seems reasonable to limit the difference in slip, between one end of the missile and the other, to the same value as the working slip in eqn. 3, so that, for example, were the slip to be 1% for each, the aim would be to accelerate the field so that the front end of the missile operated at 1.5% and the back end at 0.5%. The mean thrust would approximate very closely to that on a missile with a slip of 1% all along its length.

If it is argued that this leads to overheating of the front end of the missile, due allowance could be made in the appropriate temperature rise equation. Indeed, with all the equations developed here, refinements could be added for neglected aspects, most of which would lead to reduced ultimate speeds. It is the prime aim of the paper to set down only the most basic constraints in a form simplified sufficiently to give an overall 'feel' for the absolute maxima of velocities attainable by making the equations capable of being manipulated algebraically, without resorting to such techniques as complex layer theory. In this way the general dependency of ultimate velocities on the various parameters may be more readily determined.

It is clear that secondary cylinders should not be reduced in length below p, in which case, in the worked example above, a 2.0 cm pole pitch at 1000 m/s involves a frequency of 25 kHz. Since the pole pitch could not be increased beyond this velocity, the frequency must ultimately rise to 250 kHz at 10000 m/s, when the skin depth in copper is only 0.14 mm. Missiles of this order of wall thickness are certain to be crushed inwards by the repulsive normal forces, at less than the specific accelerating force that is required.

It could be argued that the fault lay in having decided to use a constant acceleration, which resulted in the differential slip being given by acceleration/v^2 per metre, with acceleration fixed at $v_m^2/2L$, where L is the track length. This results in the length of the secondary (and therefore the pole pitch) being given by $(v/v_m)^2\ 2Ls$, where s is the fractional slip. If instead therefore we elected to keep (dv/dx) constant ($= C$) we would have $v_m = CL$, so that the differential slip in a missile of length l is

$$s = l(dv/dx)/v = CL/v, \quad \text{or} \quad l = (v/v_m)Ls$$

which means that for a 1% slip, and a 1.0 cm missile one could use the track down to 100 m/s, or have a 10 cm long missile down to 1000 m/s. However, the required acceleration now increases linearly with v and reaches v_m^2/L at the high-end speed. This is double the requirement of the original case, but it is a relatively small price to pay for achieving a gain of five in the relationship between s and l.

Pursuing the argument even further, and keeping $(dv/dx)(l/v)$ constant would enable virtually the whole track to be used, starting from rest in a field of fixed speed v_0 until 1% slip was attained, the field thereafter accelerating at constant $(dv/dx)/v$ up to v_m such that $\log(v_m/v_0) = sL/l$ but at the expense of making the maximum acceleration equal to sv_m^2/l so that $v_m =$ 10000 m/s and $L = 100$ m, makes the maximum acceleration for a 1 m-long secondary and 0.01 slip the same as for the constant (dv/dx) case. However, a 1 m-long secondary would only allow a starting speed of 10000/ε = 3678 m/s, which is much too fast. On the other hand, a 0.1 m-long secondary would reduce v_0 to the unnecessarily low value of 0.453 m/s at the expense of maximum acceleration of 107 m/s. Nevertheless, the constant $(dv/dx)/v$ technique has allowed a certain flexibility of choice over the length of the secondary and allows other factors, such as the skin depth and the crushing forces have their say in what otherwise might well have been impossible requirements.

5 Basic heating constraints

Our relatively simple example may now be generalised by setting out all the constraints as equations. The constant $(dv/dx)/v$ restriction works out as follows.

The maximum acceleration is A_{max}, where

$$A_{max} = sv_m^2/l \tag{4}$$

The minimum field velocity

$$v_0 = v_m \varepsilon^{-sL/l} \tag{5}$$

where s is the slip derived from eqn. 3. It is now necessary to specify how low v_0 must be so that the acceleration from rest up to v_0 does not seriously add to the secondary losses that are about to be incurred during the remainder of the journey. It is reasonable to allow these initial losses to be 10% of those in the accelerated part of the field so that $mv_0^2/2 = 0.1\ s\ v_m^2/2$, or

$$v_0/v_m = \sqrt{(0.1s)} = (1/v_m)\sqrt{0.2\,ST} \tag{6}$$

Combining eqns. 3 to 6 gives

$$l = 2STL/v_m^2 \log(v_m/v_0) \tag{7}$$

$$A_m = (v_m^2/L)\log(v_m/v_0) = 2STl \tag{8}$$

6 Acceleration and crushing forces

To eqns. 1 to 8 must now be added an equation for the specific force $B_g \times J_2$, where J_2 is the secondary current loading. Now $B_g = B_c(r/2p)$ and $B_c = \mu_0 J_\phi$, where J_ϕ is the magnetising current loading, which is itself related to J_2 as $J_2 = sGJ_\phi$ and to the primary current loading J_1 as $J_1^2 = J_2^2 + J_\phi^2$. Thus

$$\text{specific force} = \mu_0 J_2^2 r/2psG \tag{9}$$

The acceleration is the total force divided by the secondary mass. The total force is the specific force multiplied by the surface area of the secondary tube while the mass is the thickness of the tube multiplied by its density and its surface area, so that the acceleration is

$$A = \mu_0 J_2^2 r/2ptDsG \tag{10}$$

and since $(dv/dx)/v$ is to be kept constant, acceleration is proportional to v^2 and will be maximum A_m when $v = v_{max}$ and minimum at A_0, when $v = v_0$, so that $A_0/A_m = (v_0/v_m)^2$.

The next equation relates the radial electromagnetic pressure $\mu_0 J_1 J_2$ to the mechanical strength, thus

$$E = \mu_0 J_1 J_2(r/2t) \tag{11}$$

At this point it is vital to note that the eqns. 10 and 11 contain several factors in common, indeed if $J_1 = J_2\sqrt{(1 + 1/s^2G^2)}$ eqns. 10 and 11 can be combined to give

$$A_m = E\sqrt{\{s^2G^2 + 1\}}/pD \tag{12}$$

In the quest for higher and higher speeds in shorter and shorter distances, eqn. 12 is a clear indicator of the problems ahead. For a fixed value of sG (which itself is only mildly 'negotiable'), $A_m \propto 1/p$. Progressive reduction of p raises warning signals about two quantities, leakage flux and magnetising current, both of which are vitally dependant on the dimensions of the magnetic circuit.

7 Primary magnetic circuit

In calculating primary related quantities it is assumed that the fundamental rule for short secondary linear motors is observed in that the primary will be parallel connected.

Precise evaluation of the ratio of the leakage reactance to resistance ratio for the primary of a tubular motor is a complex calculation. But in the interests of producing simplified equations that show the dependence of velocity on fundamental parameters without sacrificing too much accuracy, reference is made to the simplified electromagnetic system shown in Fig. 5. The primary is shown as a single layer of square-section conductors of thickness y separated from the secondary tube by an annular space of radial dimension d. The reluctance of the magnetic circuit is considered to be made up of two parts:

(*a*) the path between the primary and secondary conductors, which is basically of length p and of area $2\pi r_1 d$, and

(*b*) the path outside the primary winding can vary from an equivalent airgap of p/π to zero as shown in Fig. 2. It is already clear that in this application, ratios of r/p are

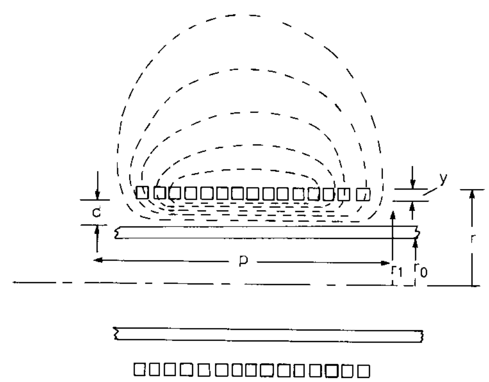

Fig. 5 *Simplified electromagnetic system*

more likely to lie in the region of 0.1 than in higher values and therefore to a very close approximation, the only reluctance is that inside the primary winding. If there are N turns per sole pitch the ratio X/R is $(\omega\mu_0/\rho_{cold})(Ndy^2/p)$. Ignoring any insulating space between primary turns $p = Ny$, whence $X/R = \omega\mu_0 dy/\rho_{cold}$. It can be argued that the primary could be multilayered in an attempt to alleviate the skin-depth limitation but the fact remains that the presence of skin effect at all in a single conductor indicates that the outer layers, if further from the interface surface than skin depth, are producing excessive amounts of leakage flux. Thus, substituting for ω from eqn. 2 (modified only by removing the slip term s since the primary winding is considered now) simplifies this to

$$X/R = 2d/y \tag{13}$$

Writing $X/R = K$, eqn. 13 becomes $y = 2d/K$.

8 Fundamental limits on v_m and L

The modified eqn. 2 then shows the maximum value of f_m is $\rho_{cold}/\pi\mu_0 y^2$, or

$$f_m = K^2\rho_{cold}/4\pi d^2\mu_0 \tag{14}$$

whence $p = v_m/2f_m$ or

$$p = 2\pi d^2\mu_0 v_m/K^2\rho_{cold} \tag{15}$$

Now eqn. 7 shows that the missile length l is inversely proportional to v_m^2, while eqn. 15 shows p proportional to v_m. Since l cannot be less than p, there is a limit on v_m set by eqns. 7 and 15 which is

$$v_m^3 \log(v_m/v_0) = 2STLK^2\rho_{cold}/2\pi\mu_0 d^2 \tag{16}$$

This limitation is not likely to be as severe as some others. For example, at $K = 6$, $d = 2$ mm, $L = 1000$ m, $v_m = 4000$ m/s, but if L is increased to 7000 m, the limit on v_m is raised to 9000 m/s. Using eqns. 8 and 12 for A_m, and substituting for p from eqn. 15 gives

$$v_m^3 \log(v_m/v_0) = LEK^2\rho_{cold}/2\pi D\mu_0 d^2\sqrt{(s^2G^2+1)} \tag{17}$$

This constraint alone sets a limit on what can be asked for by way of possible values of v_m and L. For example, were a limit to be set on X/R at say 6, and were it to be considered practical to reduce d to a minimum value of, say, 2 mm, then v_m is limited to 2520 m/s. The extent to which the maximum value of v_m is sensitive to d and K may be appreciated to a first approximation by assuming that A_m is proportional to v_m^2/L from eqn. 8 (ignoring the log term as not being very potent), and by assuming the sG value constant for the moment, it follows that v_m is proportional to $\sqrt[3]{[L\{K/d\}^2]}$. K and d are therefore more potent than is L.

Fig. 6 displays what may be achieved in the way of ultimate terminal velocities. Among the features that it

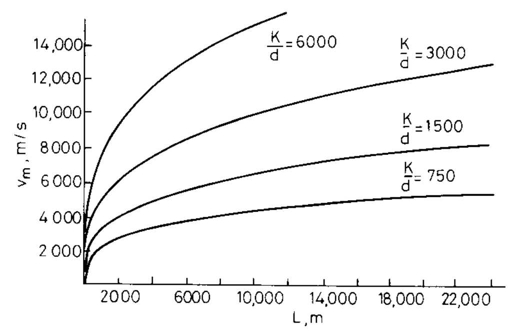

Fig. 6 *Sensitivity of v_m to d and k*

illustrates are the reasons why the Russian experiment was able to achieve accelerations of the order of 5×10^8, for they began with virtually metal-to-metal contact which just for that instant made d equal to zero.

9 Geometry of secondary member

The previous paragraph dealt essentially with matters relating to the primary, whereas G-related quantities tend to be secondary dominated. Eqn. 1 was derived under the assumption that the reluctance of the core dominated the reluctance of the magnetic circuit and this may not always be the case. Just as with the calculation of X/R, the exact evaluation of G is a complex affair so again the problem for a thick-walled cylinder is simplified in the interest of being able to outline the basic principles of design explicitly. To this end, the magnetic circuit is as shown in Fig. 5 and is to be divided into three separate regions:

Region i is the air space outside the primary winding which is equivalent to an airgap of length p/π and area $2\pi rp$, where $r = r_0 + t + d + y$.

Region ii is the distance from the outside surface of the primary to the inside surface of the secondary, a length of $(y + t + d)$, which is of area $2\pi r_g p$. Where r_g is the mean radius between r and r_0, so $r_g = [r_0 + (t + d + y)/2]$.

Region iii is the core proper, of length p and area πr_0^2.

Thus the ratio of the reluctance outside the windings to that in the core is

$$\mathscr{R}_{ext}/\mathscr{R}_{core} = r_0^2/2\pi p(r_0 + t + d + y) \quad (18)$$

while the ratio of reluctance through the windings to that in the core is

$$\mathscr{R}_W/\mathscr{R}_{core} = 2r_0^2(t + d + y)/p^2[r_0 + (t + d + y)/2] \quad (19)$$

and it is necessary to keep a check on whether each of these ratios remains small for the various values of v_m and L that are being tested.

9.1 Secondary dimensions

When it comes to deciding the wall thickness t of the secondary there are two constraints that may limit its maximum value, both concerned with skin effect:

(*a*) it may be limited by the slip frequency induced i.e. by sf_m

(*b*) it may be limited by the standstill frequency f_0 at the very start of the whole launch, where $f_0 = (v_0/v_m)f_m$.

Now $sf_m = 2STf_m/v_m^2 = ST/pv_m$, from eqn. 3, while $f_0 = f_m/v_m\sqrt{(ST/5)} = 1/p\sqrt{(ST/5)}$, from eqn. 6. Because the skin depth under (*a*) is dictated by the hot resistivity, whereas that under (*b*) relates to the cold resistivity, the two constraints are related to the value of v_m such that $v_m = \rho_{cold}\sqrt{20ST}/\rho_{hot}$.

condition (*a*) applies if $v_m < 236$ m/s

condition (*b*) applies if $v_m > 236$ m/s

Taking condition (*a*) first, eqn. 2 states that $t^2 = \rho_{hot}/\pi f_m s\mu_0$. Substituting for s from eqn 3 gives

$$t^2 = v_m^2\rho_{hot}/ST\mu_0\omega_m \quad (20)$$

Substituting for ω_m from eqn. 14 makes

$$t = (2dv_m/K)\sqrt{[\rho_{hot}/ST\rho_{cold}]} \quad (21)$$

Eqns. 10 and 11 do not enable r to be calculated yet, since both contain J^2r/t as a common factor. To evaluate r one must turn to eqn. 1 and make the initial assumption that the core reluctance dominates, so that

$$G = (\mu_0\omega/\rho_{hot})[r_0^2t/(2r_0 + t)] \quad (22)$$

It is now proposed to put $sG = 1$ on the grounds that to make $sG \ll 1$ is to incur severe penalties on input power factor and high values of J_1, whereas to make $sG \gg 1$ imposes severe limitations on A_m from eqn. 12. (With $sG = 1$, A_m is only reduced by a factor of $\sqrt{2}$ compared with its value at $sG = 0$). Eqn. 22 gives

$$r_0^2t/(2r_0 + t) = v_m^2\rho_{hot}/2ST\mu_0\omega_m$$

The similarity between the right-hand side of this equation and that of eqn. 20 is in part owing to the fact that both equations derive from reactance to resistance criteria i.e. that X/R = constant and $G = X_m/R = 1/s$. Combining the two equations gives the simple relationship

$$r_0^2t/(2r_0 + t) = t^2/2, \text{ whence } r_0 = (1 + \sqrt{3})t/2 \quad (23)$$

When condition (*b*) obtains, eqn. 20 is replaced by

$$t^2 = 2v_m\rho_{cold}/\mu_0 v_0\omega_m \quad (24)$$

and eqn. 23 is replaced by $r_0^2t/(2r_0 + t) = t^2/2\{v_m\rho_{hot}/[\rho_{cold}\sqrt{20ST}]\}$, which makes

$$r_0 = (At/2)[1 + \sqrt{(1 + 2/A)}] \quad (25)$$

where $A = v_m\rho_{hot}/\rho_{cold}\sqrt{20ST}$. Now that the missile dimensions are obtained with the assumption that the core reluctance dominates, one can return to eqns. 18 and 19 and check on the validity of the assumption. Substitution of the values appropriate to $v_m = 2600$ m/s, for example, shows that both $\mathscr{R}_{ext}$ and $\mathscr{R}_w$ contribute less than 1% to the core reluctance and since r_0, p and t are each proportional to v_m, the same will be true for all higher values of v_m.

All of these calculations of missile dimensions were made without reference to the crushing constraint, eqn. 11. Again using $sG = 1$, one can obtain a value of J_m. First, from eqn. 23, $J_m = 1.63 \times 10^7$ A/m. From eqn. 25 the value of J_m is a complex function of v_m since J_m^2 is proportional to $1/[(r_0/t) + 0.5]$ and r_0/t is multiplied by the factor $A + \sqrt{(A^2 + 2A)}/(1 + \sqrt{3})$. The value of A is given by $v_m\rho_{hot}/\rho_{cold}\sqrt{20ST} = 0.00424v_m$. As an example of the changes introduced by using condition (*b*) as opposed to condition (*a*), if $v_m = 3000$ m/s, $J_m = 8.2 \times 10^6$ A/m, whereas if $v_m = 5000$ m/s, $J_m = 6.5 \times 10^6$ A/m.

10 Primary current loading

It remains to check that it is possible to supply primary current loadings appropriate to the values of J_m derived from all the preceding equations and this will involve the temperature rise in the primary winding, a quantity that has been neglected until now.

The temperature rise T_1 is related to the current density $\boldsymbol{J}$ and to the energisation time $\boldsymbol{t}$ by the equation

$$\boldsymbol{J}^2\boldsymbol{t} = ST_1D/\rho_{cold} \quad (26)$$

derived in the companion paper [6]. ρ_{cold} is the mean resistivity of copper between 20 and 200°C; the latter temperature rise is assumed here for the purpose of the worked example. The absolute minimum energisation time is that for the secondary to pass any one point on the primary at the maximum speed i.e. l/v_m. On the assumption that generally it will be possible to reduce l to a pole pitch, the minimum energisation is therefore half a cycle, although it is obviously necessary to add several cycles to this to allow for the setting up of the flux as dictated by the value of K. The maximum current loading $J_{1(max)}$ is then given simply by $\boldsymbol{J}y$, so that

$$J_m = \boldsymbol{J}y/\sqrt{[1 + (1/sG)^2]} \quad (27)$$

A worked example gives an immediate guide to the order of the quantities involved. For a halfcycle of the supply at f_m, $\boldsymbol{J} = 3.62 \times 10^{10}$ A/m^2, so that for a primary thickness of 1.67 mm, $J_m = 6 \times 10^7$ A/m. This is an order of ten times the value found necessary for $v_m = 5000$ m/s, indicating that the switch-on time can be increased by $\sqrt{10}$, i.e. from a halfcycle to 1.5 cycles. Even this is not a 'luxurious' amount of time to allow for flux buildup, but at least the calculations in the last Section have shown that this is a diminishing problem as v_m is increased.

It remains to check whether the same thickness of primary winding will suffice at the slow speed end of the track were $v = v_0$, for theoretically it is possible to increase its thickness here from y to $y\sqrt{(v_m/v_0)}$, if necessary, to increase J_1. However, $sG_0/sG_m = \omega_0\rho_{hot}/\omega_m\rho_{cold}$ so that for $sG_0 = 1$, $G_0 = v_m\rho_{hot}/\rho_{cold}\sqrt{20ST}$ (numerically $G_0 = v_m/236$). Hence, $sG_0 = \rho_{hot}\sqrt{(ST/5)}/v_m\rho_{cold}\sqrt{20ST}$ (numerically $sG_0 = 3377/v_m$).

The numerical calculations are interesting in that they show for quite high terminal speeds, up to 3377 m/s, sG_0 is actually > 1. For higher speeds sG_0 will deteriorate, but remember that because of the technique of making

$(1/v)dv/ds$ constant, $A \propto v^2 \propto J^2$ so that in particular $J_{20}/J_{1m} = v_0/v_m$. Also, $J_{10}/J_{20} = \sqrt{(1 + 1/s^2G^2)}$ and $J_{1m}/J_m = \sqrt{2}$. Thus

$$J_{10}/J_{1m} = (v_0/v_m)\sqrt{[0.5 + 10v_m^2 \rho_{cold}^2/ST\rho_{hot}^2]}$$

Numerically, this becomes $J_{10}/J_{1m} = \sqrt{[0.0035 + (22/v_m)^2]}$ which is >1 for all high values of v_m and there is never any reason to increase the thickness of the primary winding at the low-speed end of the track.

11 Missile mass

Making reasonable approximations it is possible to express the mass of the secondary explicitly, thus: mass $= 2\pi(r_0 + t/2)tD$. Examination of the order of the quantities in eqn. 26 shows that very little error is made in writing $r_0 = At$. Substituting for t^2 from eqn. 25, f_m from eqn. 14 and $l(=p)$ from eqn. 15 and simplifying gives

$$M = 8\pi^2(d/K)^4(\mu_0 D/ST)(\rho_{hot}/\rho_{cold}^2)v_m^3 \quad (28)$$

Numerically, $M = 1490\ (d/K)^4 v_m^3$ and to set the level, for $v_m = 3000$ m/s, the missile mass is approximately 0.5 kg, rising to 148 kg at 20000 m/s.

12 Conclusions

It is clear that velocities of 20 km/s *can* be reached by induction, but only in large sizes and at the expense of enormously long tracks. The power levels to achieve such velocities in a mass (for example) of 150 kg are of the order of 6000 GW and kinetic energy of 30000 MJ.

Eqns. 1 to 28 enable ultimate velocities by induction to be evaluated for the first time in terms of a given track length and missile mass. These velocities are dictated by the five properties of copper listed in Section 14 and by the permeability of free space, by the power factor of the primary and by mechanical clearance. There is, of course, no better metal than copper.

A number of workers in the field of electromagnetic launchers have written about the possibility of shaping the flux wave into more advantageous forms than those of sinusoidal space distribution. There have been proposals to produce a shaped pulse of flux that chases the secondary down the barrel. This technique is very similar to that proposed here except that it is essential that the 'pulse' should consist of a short burst of sine waves, otherwise the temperature limitations on the secondary are aggravated, for a complex travelling field, where the spatial waveshape is nonsinusoidal, can be shown to be equivalent to a set of travelling fields superposed, whose speeds form part or all of a Fourier series. Whichever speed dictates the secondary speed, and is therefore to be seen as v_m in $(1/2)mv_m^2$, this field itself is 50% energy efficient from rest up to synchronism, (better for smaller speed ranges) but all the other fields are proportionally worse. Elliott reports [2] one tested system as having a secondary ohmic loss 1.9 times that of a sinusoidally fed system.

The work in this paper is essentially concerned with electromagnetic constraints on coil guns. Mechanical limitations and power supplies are additional subjects only touched marginally here. For example, the results displayed in Fig. 6 show the dependence on the dimension d and on the X/R value for the primary winding. Eqn. 17 shows how beneficial it would be to increase effectively, the value of E. One might, for example, consider having a solid secondary, at the expense of increasing the missile mass by a large factor ($\sim r_0/2t$), but since $r_0 \propto v_m^{3/2}$ and $t \propto v_m^{1/2}$, this effectively replaces v_m^3 by v_m^4 on the left-hand side of eqn. 17, which is a high price to pay. On the other hand, it could be argued that what is needed is to fill the core with a relatively light material that is incompressible, such as water! The combined effects of the relative thermal expansions of water and copper, the effect of pressure on the boiling point of water and the fact that the low thermal conductivity of water will only allow an extremely thin layer of the water in contact with the copper to attain the melting temperature of copper are extremely complex. There is obviously scope for a great deal more work to be done.

13 References

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14 Appendix

The important symbols in the foregoing equations and their values for copper are as shown below:

S = specific heat of copper = 385 J/kg, °C

ρ = resistivity:

$\rho_{hot} = 21.3 \times 10^{-8}$ ohm-m (just before melting)

$\rho_{cold} = 2.13 \times 10^{-8}$ ohm-m (at speed v_0)

D = density = 8.9×10^3 kg/m^3

T = melting point = 1053°C

E = yield strength = 2.2×10^8 N/m^2

μ_0 = free-space permeability = $4\pi \times 10^{-7}$ h/m