Abstract
Barrel current in a traveling wave coilgun must be commutated on and off and remain synchronized with the position of the armature coils. Brushes on the armature can be used to satisfy these two requirements. First, they provide synchronization of the currents by being physically attached to the armature coil. Second, they provide a means of shorting the turns of the barrel so that motion induced commutation can take place.

Motion induced commutation is the process where the current in a shorted barrel turn is caused to rise or fall because of the voltage induced by the moving armature. This change in barrel current must take place during the time the turn is shorted by the brush. Residual current in the trailing edge of the brush must be made small at the instant the circuit is opened to minimize arcing.

A set of design criteria are presented for designing a brush commutated coilgun so as to achieve the required synchronous commutation. The design variables include the barrel coil configuration and its pitch, the armature coil configuration and its number of turns, and the brush length. The number of turns in the barrel undergoing commutation at the same instant in time is directly related to the length of the brush.

The results of applying the design criteria to a specific design are presented.

Introduction
Coilguns require less current than railguns to produce the same force because of the multiplication effect of the coil turns [1]. Coilguns, however, unlike railguns, require commutation of the current in the barrel turns. Most of the research on brush commutated coilguns seems to have been confined to the last ten years.

Early research concentrated on the receding front coilguns where commutation was required to turn off the current [2, 3]. These first attempts explained the commutation process in terms of the concept of motion induced voltages and gave explicit mathematical formulas for multiple turn commutation. These early formulas were valid for one and two turn commutation only.

Other attempts to describe brush commutation in coilguns analyzed the process as an energy transfer problem between stationary inductors [4, 5]. This approach does not adequately consider the motion induced voltage and gives incorrect results [6]. Brush commutation has been studied in some depth recently [7]. This latter report forms the basis for the present paper.

Commutation
Some degree of commutation will always occur. The task is to design the coilgun for efficient commutation. To illustrate this point, consider the rear portion of the activated section of the barrel. The goal is to reduce the current in the last turn to zero while it is shorted. Even if the current in the last turn of these energized turns is not fully commutated, the current will snap to zero if it is sufficiently small. Some of the stored magnetic energy associated with the commutating turn will be lost in the spark but most of it will be transferred to the rest of the circuit through magnetic coupling. If the current is not sufficiently small at the instant the circuit is opened, the current will not snap to zero, but continue to burn. The resulting arc would, besides damaging the barrel, prevent all the remaining turns from fully opening and reduce the force imbalance that drives the armature.

The commutation process may be divided into three steps: close switch, commutate current, open switch. The commutation process begins by electrically isolating the turn to be commutated. For brush commutation the leading edge of the brush acts as a closing switch and isolates the turn by shorting it. Next, the current in the shorted turn is turned on or off by by some type of driving voltage. For brush commutation the movement of the armature causes the change in flux required to induce a voltage in the shorted turn. To maintain synchronization the turn must be returned to the circuit in one cycle. For brush commutation the trailing edge of the brush acts as an opening switch and returns the turn to the circuit by removing the short.

Design Criteria
One of the chief design requirements for high speed coilguns is efficient commutation. On the basis of a qualitative analysis similar to the one presented above the authors suggest the following three criteria for efficient commutation in brush commutated traveling wave coilguns. The first criterion concerns electrical breakdown before the commutation starts. The second criterion concerns changing the current while the turn is shorted. The third criterion concerns electrical breakdown after the commutation is complete.

1. Pre-Commutation Criterion:
Keep the induced voltage at the leading edge of the brush below the electrical breakdown voltage.
2. Commutation Rate Criterion:
With the induced voltage reduce the current to zero in the rear and increase the current to its full value in the front during the time the turn is shorted.

3. Post-Commutation Criterion:
Keep the induced voltage at the trailing edge of the brush low enough to avoid electrical breakdown or re-strike and extinguish any arc that forms.

This paper has two major objectives. One objective is to present a mathematical formulation of the above commutation rate criterion for single turn and multiple turn commutation. The other two criteria impose additional restrictions on the coligun design but are beyond the scope of this paper.

The second objective is to give examples illustrating the current waveforms in the shorted turns during the commutation cycle. The current waveforms are basic to any further analysis. Once they have been determined, one can calculate the turn-to-turn voltage and the magnetic forces.

**Traveling Wave Coligun**

A traveling wave coligun consists of a barrel coil, two armature coils, feed rails and brushes. The relationship of these components is illustrated in Fig. 1 and a simplified electrical schematic is given in Fig. 2.

The gun is fed by a pair of feed rails. Current passes from the feed rail into the rear armature coil through a feed-brush, through the armature coil and then into the trailing end of energized section of the barrel coil through a commutating-brush. Both types of brushes are rigidly attached to the armature. The current energizes a section of the barrel coil and then pass out of the leading edge of this section into the front armature coil through another commutating-brush. The current next passes through the front armature coil, in the opposite direction from that of the rear armature coil. The current goes into a second feed rail through another feed-brush. The helical coil of the barrel is electrically insulated on its inside diameter except were it is fed current from the brushes.

The driving force for the coligun is the JxB magnetic force on each of the armatures due to the field produced by the energized section of the barrel coil. The magnetic force on the front armature coil is one of repulsion while that on the rear armature coil is attraction.

The induced voltage producing commutation and the accelerating magnetic force must work together. There is only one possible spatial arrangement under which this happens: the shorted turns must be located in front of the armature coil. This requirement determines the location of the brushes. They must be located so that the turns they short are located physically ahead of the armature. These relationships are illustrated in Fig. 1.

The current distribution within the barrel coil, for two turn commutation, is shown in Fig. 3. As the armature coils advance one pitch the current in the commutating turns goes from an initial to a final value. The final value of the current in one commutating turn is equal to the initial value of the neighboring turn behind it. This process repeats for every pitch that the armature moves until the armature reaches the muzzle.

The coligun configuration given in Table I was chosen to illustrate the design criteria and waveforms. The current in the barrel turns and the current in the armature coil is assumed to be uniformly distributed. The one millimeter radial thickness of the barrel

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**Fig. 1.** Traveling wave coligun configuration with direction of current indicated.

**Fig. 2.** Configuration of barrel turns and armature coils for a traveling wave coligun just after the rear of the brush has opened a shorted turn and the front of the brush has closed a shorted turn.
Fig. 3. Current distribution among the barrel coil turns for two turn commutation.

The circuit equation for a shorted turn is:
\[
\frac{d(I)}{dt} + IR + \frac{d(M_{1}I_{0})}{dt} + \frac{d(M_{2}I_{0})}{dt} = 0
\]

where
- \( I = \) self inductance of shorted barrel turn,
- \( I = \) current in shorted turn,
- \( R = \) resistance of shorted turn circuit,
- \( M_{1} = \) mutual inductance between shorted turn and armature coil,
- \( M_{2} = \) mutual inductance between shorted turn and energized section of barrel coil that is in series with armature coil,
- \( I_{0} = \) current in coligun circuit.

The first term is due to the self inductance of the turn being turned on and off. The second term contains all of the resistance of the shorted turn circuit (barrel turn, brush contact, and brush bulk). The third term is the motion induced voltage due to the moving armature. The plus (negative) sign is for a circuit in which the current in the armature and the current in the shorted turn are in the same (opposite) direction. The induced voltage is positive for a shorted turn being turned off and negative for a shorted turn being turned on. The fourth term is due to changes in the current in the coligun circuit.

There could be other terms in the equation. If the two armature coils are close together, there will be another induced voltage term due the coupling between the shorted turns and the other armature coil. There are ways to turn the current off and on faster than that provided by the motion induced voltage. One could use additional switches and an external voltage source like a capacitor [8].

The fundamental aspect of commutation will be seen more easily by dropping non-essential terms. If the two shorted turns are spaced more than two barrel coil diameters apart then there will be negligible (less than 2%) coupling. The current in the armature coil and the fully energized portion of the barrel coil is the same because the two coils are in series. If this current is constant over the commutation period, then the last term in the equation can be neglected. This is supported by the fact that in typical coliguns there are greater than 100 barrel turns and less than 50% change in input current. Resistance has the effect of aiding the turn off commutation and hindering the turn on commutation. When the time constant of the shorted turn is much greater than the time to commutate the turn, then resistance can be neglected. For typical coliguns this happens when armature velocities exceed 100 m/sec. When designing coliguns with a low muzzle velocity one may need to keep a...
resistance term. In high velocity coilguns where the muzzle velocity exceeds 1 km/sec, resistance will affect commutation at the breech only.

The circuit equation for one turn commutation now reduces to:

$$\frac{d}{dt} \left[ L I + M_a I_0 \right] = 0$$

In this form it is clear that, for the conditions under consideration, flux is conserved during the commutation process. Integrating this equation as the armature advances one pitch will give the change in barrel coil current during commutation.

$$I = \pm I_0 \left[ -\frac{\Delta M_a}{L} \right]$$

where

$$\Delta M_a = N \left[ M(z_2) - M(z_1) \right]$$

and

$$N = \text{number of turns in armature coil},$$
$$M(z) = \text{mutual inductance function; single turn effective mutual inductance between armature coil and shorted turn as a function of } z,$$
$$z_1/z_2 = \text{axial distance separating shorted turn and armature coil, initial and final values respectively.}$$

Using the commutation rate criterion the required change in current is \( -I_0 \) and \( +I_0 \), for the rear and front coils respectively. Thus, the number of armature turns required to accomplish full commutation solely by the motion induced voltage term is given by:

$$N = \frac{L}{M(z_2) - M(z_1)}$$

The current in the front shorted turn as a function of position is:

$$I = I_0 \left[ -\frac{N}{L} \left( M(z) - M(z_1) \right) \right]$$

The current in the rear shorted turn is:

$$I = I_0 \left[ 1 - \frac{N}{L} \left( M(z) - M(z_1) \right) \right]$$

The current in the front shorted turn was calculated for the coilgun configuration given in Table I with two different values for the offset (zero and one pitch). The results are shown in Fig. 5 as a function of time. The normalized time is defined as the distance moved by the armature divided by one pitch length.

**Multiple Turn Commutation**

A mathematical formula for the commutation rate criterion is derived in this section for multiple turn commutation. As with single turn commutation, once the coilgun configuration is fixed, the chief design variables controlling motion induced commutation are the number of turns in the armature and the brush offset. The design criterion specifies what this number should be to obtain full commutation. As before the armatures in the traveling wave coilgun are assumed to be widely spaced. The circuit diagram showing the relative positions of the barrel turns and armature coils for a multiple turn commutation traveling wave coilgun is shown in Fig. 2.

For simplicity it will be assumed that both front and rear armature coils have the same configuration and brush offsets. As before, the resistance and any changes in the armature current will be neglected.

The simplified circuit equation in matrix form for both front and rear commutation regions is:

$$\frac{d}{dt} \left[ L K \cdot I + N M I_0 \right] = 0$$

where

$$L = \text{self inductance of a single shorted turn},$$
$$K = \text{matrix of coupling coefficients for the shorted turns in the commutating section,}$$
$$I = \text{array of currents for the shorted turns,}$$
$$N = \text{number of turns in the armature coil,}$$
$$M = \text{array of mutual inductances for the armature coil and shorted turns,}$$
$$I_0 = \text{current in the armature coil.}$$

The positive (negative) sign indicates that the current in the armature coil is in the same (opposite) direction as the current in the barrel coil. Hence, the positive (negative) sign applies for the rear (front) armature and its barrel turns.

The matrix of coupling coefficients can be represented as:

$$K = \begin{bmatrix}
1 & k_1 & k_2 & \cdots & k_{n-1} \\
k_1 & 1 & \vdots & \vdots & \\
& k_2 & 1 & \ddots & \\
& & \ddots & \ddots & 1 \\
& & & k_{n-1} & 1
\end{bmatrix}$$
where \( k_0 \) is the coupling coefficient between a shorted turn and one of its nearest neighbors, \( k_n \) is the coupling coefficient between a shorted turn and one of its next nearest neighbors, etc.

The array of currents can be represented as:

\[
I(z) = \begin{bmatrix}
I_1(z) \\
I_2(z) \\
\vdots \\
I_N(z)
\end{bmatrix}
\]

where \( I_j(z) \) is the current in the \( j \)th shorted turn as a function of the distance \( z \) between the armature coil and the first shorted turn. If \( w \) is the barrel pitch, then \( z \) is restricted to region \( z_0 < z < z_0 + w \).

The array of mutual inductances can be represented in a similar fashion with \( M_j \) as the mutual inductance between the \( j \)th shorted turn and the armature coil. Note that \( M_j(z) \) is equal to \( M(z+jw) \), where \( w \) is the pitch.

As before, flux is conserved during one pitch of the commutation cycle and the above equation can be integrated to obtain the incremental change in current for the shorted turns:

\[
L K \cdot \Delta I = N \Delta M I_0 = 0
\]

The integration is performed from \( z = z_0 + w \) to \( z = z_0 \). Solving for the change in current, \( \Delta I \), yields

\[
\Delta I = \frac{\mp I_0}{L} K^{-1} \Delta M
\]

where

\[
K^{-1} = \text{inverse of the matrix } K
\]

\[
\Delta M_j = M( (j-1)w + z_0 ) - M( jw + z_0 )
\]

\( j = 1, 2, 3 \ldots n \)

\( w = \text{axial distance between turns} = \text{pitch} \),

\( z_0 = \text{axial offset of brush} \).

The commutation rate criterion, for \( n \)-turn commutation, is this: when the armature coil moves a distance equivalent to \( n \) pitches, the current in a shorted turn is continuous and its total change is \( I_n \). Continuity requires that the initial value of \( I_n(z) \) equal the final value of \( I_{n+1}(z) \). The sum of the current increments add up to the total change for one turn.

\[
\sum_j \Delta I_j = \pm I_0
\]

The summation is over all \( n \) shorted turns. The positive (negative) sign applies for the front (rear) armature and its barrel turns. The current is being turned off (on) in the shorted turns for the rear (front) coil.

The value of \( N \) that satisfies the above two equations is the number of turns required in the armature coil in order that full commutation can be accomplished solely through motion induced voltage. The result is:

\[
N = \sum_j \left[ K^{-1} \Delta M \right]_j
\]

where

\[
\left[ K^{-1} \Delta M \right]_j = \text{the } j \text{th element in the array resulting from the matrix multiplication of } K^{-1} \text{ and } \Delta M.
\]

The initial values of the individual shorted turn currents can be determined from recurrence relations. Let \( z_1 = z_0 + w \). For the front set of shorted turns:

\[
I_1(z_1) = 0
\]

\[
I_j(z_1) = I_{j+1}(z_1) + \Delta I_{j+1} ; j = 1, 2 \ldots n-1
\]

For the rear set of shorted turns:

\[
I_1(z_1) = I_0
\]

\[
I_j(z_1) = I_{j+1}(z_1) - \Delta I_{j+1} ; j = 1, 2 \ldots n-1
\]

The evolution of the current in the individual shorted turns can now be written in a form similar to that for single turn commutation. For the front set of shorted turns:

\[
I_j(z) = I_j(z_1) + I_0 \frac{N}{L} \left[ K^{-1} \Delta M \right]_j
\]

where

\[
\Delta M_j = M( jw + z_0 ) - M( jw + z_1 )
\]

and \( z_0 < z < z_1 \).

For the rear set of shorted turns:

\[
I_j(z) = I_j(z_1) - I_0 \frac{N}{L} \left[ K^{-1} \Delta M \right]_j
\]

The currents in the front set of shorted turns were calculated for the colgum configuration given in Table I. The results are presented in Figs. 6 and 7. The mutual inductance function is the same that use for the single turn example. The coupling coefficients were calculate using the same filamentary model.

In Fig. 6 the current in a shorted turn is shown for two-turn commutation and two values of the brush offset distance, \( z_0 \). Note that, near the end of the commutation cycle for the zero offset case, the current momentary exceeds the operating current \( I_0 \).

Fig. 7 shows the front commutating currents with one to five turns being commutated. For ease of comparison, the currents are all plotted so that their commutation is completed at the same point. The length of the region being commutated is the same as the length of the brush. The length of the brush is equal to the barrel pitch multiplied by the number of turns being commutated. From Figure 7 it is clear that it is not the number of turns (five for this example) being commutated, but the commutation length that is important. There is a point beyond which further increase in the length of the commutation region has no effect on the current waveform. This point occurs when the commutation length is equal to about half the barrel coil diameter.
Conclusions

The process of commutation in the barrel coil turns was analyzed for a traveling wave coilgun. It was found that the commutating turns must be located ahead of the midplane of the armature coils in order that the accelerating force and the commutation process work in unison.

Three design criteria were proposed to insure efficient commutation. One of these, the commutation rate criterion, was converted into a mathematical formulation for single turn and multiple turn commutation. The resulting formulas relate the number of turns in the armature with the number of turns being commutated, the brush offset and all the other dimensions of the coilgun as expressed through the inductance terms.

Formulas for calculating the time evolution of the individual commutating currents were presented and illustrated with examples.

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References


